Bit Interleaved Coded Modulation with Space Time Block Codes for OFDM Systems

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*Abstract***— Wireless systems often implement one or more types of diversity in order to achieve reliable communication. Different types of diversity techniques such as temporal, frequency, code, and spatial have been developed in the literature. In addition to the destructive multipath nature of wireless channels, frequency selective channels pose intersymbol interference (ISI) while offering frequency diversity for successfully designed systems. Orthogonal frequency division multiplexing (OFDM) has been shown to combat ISI extremely well by converting the frequency selective channel into parallel flat fading channels. On the other hand, bit interleaved coded modulation (BICM) was shown to have high performance for flat fading Rayleigh channels. Combination of BICM and OFDM was shown to exploit the diversity that is inherited within the frequency selective fading channels. In other words, BICM-OFDM is a very effective technique to provide diversity gain, employing frequency diversity. Orthogonal space-time block codes (STBC) make use of diversity in the space domain by coding in space and time. Thus, by combining BICM-OFDM and STBC, diversity in frequency and space can be taken advantage of. In this paper we show and quantify both analytically and via simulations that for frequency selective fading channels, BICM-STBC-OFDM systems can fully and successfully exploit the frequency and space diversity to the maximum available extent.**

I. INTRODUCTION

Problems due to multipath and interference from other users in wireless channels are well known. In order to alleviate these problems, a number of diversity techniques have been proposed. There are examples of such techniques in time, frequency, space, and code domains.

An important way to achieve this diversity for coded systems was invented by Zehavi who showed that the code diversity could be improved by bit-wise interleaving [1]. Using an appropriate soft-decision bit metric at a Viterbi decoder, Zehavi achieved a code diversity equal to the smallest number of distinct bits, rather than channel symbols, along any error event. On the other hand, the order of diversity for any coded system with a symbol interleaver is the minimum number of distinct symbols between codewords. This difference between bit-wise interleaving and symbol interleaving results in improved performance for BICM over a fading channel. Following Zehavi's paper, Caire *et al* [2] presented the theory behind BICM. Their work provides tools to evaluate the

performance of BICM with tight error probability bounds, and design guidelines.

However, when there is frequency selectivity in the channel, the design of appropriate codes becomes a more complicated problem due to the existence of intersymbol interference (ISI). On the other hand, frequency selective channels offer additional frequency diversity [3], [4], and carefully designed systems can exploit this property. OFDM can be used to combat ISI and therefore can simplify the code design problem for frequency selective channels. It is shown in [5] that the combination of BICM and OFDM systems can achieve the full diversity order of L for L-tap frequency selective channels.

In recent years deploying multiple transmit antennas has become an important tool to improve diversity. The use of multiple transmit antennas allowed significant diversity gains for wireless communications. In general, spatial diversity systems are called space-time (ST) codes and some important results can be listed as [6], [7], [8], [9], [10]. In these papers the multi input multi output (MIMO) wireless channel is assumed to be flat fading. If the channel is frequency selective, then carefully designed space-time-frequency coded systems have been proposed to exploit the diversity order in space and frequency, [11], [12], [13], [14], [15], [16]. Out of these papers [15] combines space time block codes (STBC) of [7] and [8] with bit interleaving for OFDM systems. Reference [16] uses BICM-OFDM directly with multiple antennas and without external STBC to achieve higher data rate in the cost of lower diversity.

In [5] it is shown that BICM-OFDM can successfully exploit the frequency diversity for single antenna systems. Here, STBC is added to BICM-OFDM to exploit the diversity not only in frequency but also in space to its maximum extent. The results of [5] are used as a basis to carry out the analysis for BICM-STBC-OFDM. The reader is urged to note that unlike [15], we *formally prove* that BICM-STBC-OFDM systems can achieve the full diversity order that can be offered by the channel. In addition to analysis, through simulations, the performance of BICM-STBC-OFDM as compared to [6] and [10] with OFDM are illustrated. We will show that for systems employing N transmit and M receive antennas, over L-tap frequency selective channels, BICM-STBC-OFDM can achieve the maximum diversity order of NML.

The rest of the paper is organized as follows. We present brief overviews of STBC and BICM in Sections II and III, respectively. The system model for BICM-STBC-OFDM is introduced in Section IV. The diversity order of BICM-STBC-OFDM system over frequency selective fading channels is given in Section V. Simulation results supporting our analysis are presented in Section VI. Finally, we end the paper with a brief conclusion in Section VII where we restate the important results of this paper.

II. SPACE TIME BLOCK CODES (STBC)

Complex orthogonal space time block codes [8] are considered in this paper. For N transmit antennas, S/T rate STBC is defined as the complex orthogonal block code which transmits S symbols over T time slots. Code generator matrix G_{STN} is a $T \times N$ matrix and satisfies [8]

$$
G_{STN}^H G_{STN} = \kappa (|x_1|^2 + |x_2|^2 + \ldots + |x_S|^2) I_N \quad (1)
$$

where κ is a positive constant and $\{x_i\}_{i=1}^S$ are the complex symbols transmitted in one STBC codeword. For example, Alamouti code [7] is a rate one STBC given as

$$
G_{222} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}
$$
 (2)

III. BIT-INTERLEAVED CODED MODULATION (BICM)

BICM can be obtained by using a bit interleaver, π , between an encoder for a binary code C and a memoryless modulator over a signal set $\chi \subseteq \mathbb{C}$ of size $|\chi| = M = 2^m$ with a binary labeling map $\mu : \{0, 1\}^m \to \chi$. Gray encoding is used to map the bits onto symbols and plays an important role in BICM's performance for non-iterative decoding, [2]. It is shown in [17] that the capacity of BICM is surprisingly close to the capacity of multilevel codes (MLC) scheme if and only if Gray labeling is used. Moreover, Gray labeling allows parallel independent decoding for each bit. In [17] it is actually recommended to use Gray labeling and BICM for fading channels. If set partition labeling or mixed labeling is used, then an iterative decoding approach should be used to achieve high performance [18]. Note that, due to the ability of independent parallel decoding of Gray labeling, iterative decoding does not introduce any performance improvement [18]. Therefore, non-iterative maximum likelihood (ML) decoding is considered in this paper.

During transmission, the code sequence \overline{c} is interleaved by π, and then mapped onto the signal sequence $\mathbf{x} \in \mathcal{X}$. The signal sequence \overline{x} is then transmitted over the channel.

The bit interleaver can be modeled as $\pi : k' \to (k, i)$ where
denotes the original ordering of the coded bits $c_{i,k}$ denotes k' denotes the original ordering of the coded bits $c_{k'}$, k denotes the time ordering of the signals x_k transmitted, and i indicates the position of the bit $c_{k'}$ in the symbol x_k .

Let χ_b^i denote the subset of all signals $x \in \chi$ whose label has the value $b \in \{0, 1\}$ in position *i*. Then, the ML bit metrics are given by [2]

Fig. 1. Block diagram of BICM-STBC-OFDM

$$
\lambda^{i}(y_{k}, c_{k'}) = \begin{cases} \max_{x \in \chi_{c_{k'}}^{i}} \log p_{\theta_{k}}(y_{k}|x), & \text{perfect CSI} \\ \max_{x \in \chi_{c_{k'}}^{i}} \log p(y_{k}|x), & \text{no CSI} \end{cases}
$$
(3)

where θ_k denotes the channel state information (CSI) for the time order k.

Following (3), the bit metrics for the flat fading Rayleigh channels can be calculated using the ML criterion with CSI as [1]

$$
\lambda^{i}(y_{k}, c_{k'}) = \min_{x \in \chi^{i}_{c_{k'}}} \|y_{k} - \rho x\|^{2}
$$
 (4)

where ρ denotes the Rayleigh coefficient and $||(\cdot)||^2$ represents the squared Euclidean norm of (\cdot) .

The ML decoder at the receiver can make decisions according to the rule

$$
\underline{\hat{\mathbf{c}}} = \arg\min_{\underline{\mathbf{c}} \in \mathcal{C}} \sum_{k'} \lambda^i(y_k, c_{k'}).
$$
 (5)

IV. SYSTEM MODEL OF BICM-STBC-OFDM

In BICM-STBC-OFDM, a rate S/T STBC is used to code the tones of an OFDM symbol across time and space, and BICM is applied for coded modulation. One OFDM symbol has K tones, where each tone is a complex constellation point. STBC for the tone k is given by the $T \times N$ matrix $C(k) = G_{STN}(x_1(k),...,x_S(k))$, which is calculated by applying the symbols $x_1(k),...,x_S(k)$ to the STBC generator matrix G_{STN} .

The output bits of a convolutional encoder are interleaved within T OFDM symbols to avoid extra delay requirement to start decoding at the receiver. After interleaving, the output bit $c_{k'}$ is mapped onto the tone $x_s(k)$ at the *i*th bit location, where $1 \leq s \leq S$. It is assumed that an appropriate length of cyclic prefix (CP) is used for each OFDM symbol. As a result, the received signal for each tone over M receive antennas is given by the $T \times M$ matrix

$$
R(k) = C(k)H(k) + N(k)
$$
\n(6)

where $N(k)$ is a $T \times M$ complex additive white Gaussian noise with zero mean and variance $N_0 = N/SNR$, and $H(k)$ is given by

Normalized 16QAM Signal Constellation with Gray Encoding					Normalized 16QAM Signal Constellation with Gray Encoding, χ^0				Normalized 16QAM Signal Constellation with Gray Encoding, χ^0 .				
зþ	0010	0110	1110	1010		0010	0110				1110	1010	
	0011	0111	1111	1011	0.8	0011	0111		0.8		1111	1011	
	0001	0101	1101	1001	-0.8	0001	0101		-0.8		1101	1001	
	0000 ×. 4	0100	1100	1000	-1 -1	0000	0100 1.5	GB	-1	-0.8	1100 G.B.	1000	
(a) χ						(b) χ_0^0				(c) χ_0^1			

Fig. 2. Gray encoded 16 QAM constellation

$$
H(k) = \begin{bmatrix} H_{11}(k) & H_{12}(k) & \cdots & H_{1M}(k) \\ H_{21}(k) & H_{22}(k) & \cdots & H_{2M}(k) \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1}(k) & H_{N2}(k) & \cdots & H_{NM}(k) \end{bmatrix}_{N \times M}
$$

$$
H_{nm}(k) = \underline{\mathbf{W}}^{H}(k)\underline{\mathbf{h}}_{nm}
$$

$$
\underline{\mathbf{W}}(k) = [1 \quad W_{K}^{k} \quad \cdots \quad W_{K}^{(L-1)k}]^{H}, \text{ where } W_{K}^{k} \stackrel{\triangle}{=} e^{-j\frac{2\pi}{K}}
$$

$$
\underline{\mathbf{h}}_{nm} = [h_{nm}(0) \quad h_{nm}(1) \quad \cdots \quad h_{nm}(L-1)]^{T} \tag{7}
$$

where \underline{h}_{nm} represents the L tap frequency selective channel from the transmit antenna n to the receive antenna m . Each tap is assumed to be statistically independent and modeled as zero mean complex Gaussian random variable with variance $1/L$. It is assumed that the taps are spaced at integer multiples of the symbol duration, which is the worst case scenario in terms of designing full diversity codes [19]. The fading model is assumed to be quasi-static, i.e., the fading coefficients are constant over the transmission of one packet, but independent from one packet transmission to the next. Note that, the average energy transmitted from each antenna at each subcarrier is assumed to be 1. Then, with the given channel and noise models, the received signal to noise ratio is SNR.

V. DIVERSITY ORDER OF BICM-STBC-OFDM

In this section, by calculating the pairwise error probability (PEP), we will show that BICM-STBC-OFDM can achieve the maximum achievable diversity order of NML. Assume that binary codeword $\frac{c}{c}$ is sent and $\frac{c}{c}$ is detected. Then, the PEP is written as

$$
P(\underline{\mathbf{c}} \to \underline{\hat{\mathbf{c}}}|\mathbf{H}) = P\left(\begin{array}{c}\sum_{k'} \min_{x_s \in \chi_{c_{k'}}^i} \|R(k) - CH(k)\|_F^2 \geq \\ \sum_{k'} \min_{x_s \in \chi_{c_{k'}}^i} \|R(k) - \hat{C}H(k)\|_F^2 \end{array}\right)_{(8)}
$$

where $\|(\cdot)\|_F^2$ denotes $\|(\cdot)\|_F^2 = Tr\{(\cdot)^H(\cdot)\}$ (square of the Frobenius norm of (.)) and C and C denote the two distinct Frobenius norm of (\cdot) , and C and C denote the two distinct STBC codewords.

Note that $||R(k)-CH(k)||_F^2$ provides S equations to decode
symbols within STBC C [81. [91. As mentioned in Section S symbols within STBC C [8], [9]. As mentioned in Section IV, the output bit $c_{k'}$ is mapped onto the *i*th bit of $x_s(k)$. So the bit metric for each $c_{k'}$ is found by minimizing the sth equation given by $||R(k) - CH(k)||_F^2$ with respect to $x_s \in \mathbb{R}^d$ $\chi^{\dot{i}}_{c_{k'}}$.

For a k_0/n_0 convolutional code with the minimum Hamming distance d_{free} , the worst case scenario in (8) simplifies to a summation for only d_{free} terms. Note that, for the d_{free} points $\hat{c}_{k'} = \bar{c}_{k'}$, where (\cdot) denotes the binary complement of (c). Also, $\chi^i_{c_{k'}}$ and $\chi^i_{c_{k'}}$ are complement sets of constellation points within the signal constellation set χ (see Figure 2 for points within the signal constellation set χ (see Figure 2 for 16 QAM example). Let's denote

$$
\tilde{C}(k) = \arg \min_{\substack{C = G_{STN}(x_1, ..., x_S) \\ s.t. x_s \in \chi_{c_{k'}}^{i}}} \|R(k) - CH(k)\|_{F}^{2}
$$
\n
$$
\hat{C}(k) = \arg \min_{\hat{C} = G_{STN}(x_1, ..., x_S) } \|R(k) - \hat{C}H(k)\|_{F}^{2}
$$
\n(9)

 $\tilde{C}(k)$ and $\tilde{C}(k)$ are distinct two matrices whose sth elements are from $\chi^i_{c_{k'}}$ and $\chi^i_{\bar{c}_{k'}}$, respectively. For convolutional codes, d_{free} distinct bits between any two codewords occur in consecutive trellis branches. The bit interleaver can be designed such that consecutive $\left[d_{free}/n_0\right]$ bits are mapped onto $\left[d_{free}/n_0\right]$ different tones of an OFDM symbol. This guarantees that there exists d_{free} distinct pairs of $(\tilde{C}(k), \tilde{C}(k))$ for PEP analysis. Also note that, $||R(k) - C(k)H(k)||_F^2 \ge$
 $||R(k) - \tilde{C}(k)H(k)||_F^2$ and $C(k) \neq \hat{C}(k)$ for the dx $||R(k) - \tilde{C}(k)H(k)||_F^2$, and $C(k) \neq \tilde{C}(k)$ for the d_{free}
matrices under consideration. Then (8) can be rewritten as matrices under consideration. Then, (8) can be rewritten as

$$
P(\underline{\mathbf{c}} \to \underline{\hat{\mathbf{c}}}|\mathbf{H}) = P\left(\sum_{k,d_{free}} \frac{\|R(k) - \tilde{C}(k)H(k)\|_F^2}{\|R(k) - \hat{C}(k)H(k)\|_F^2} \ge 0\right)
$$

$$
\le P\left(\sum_{k,d_{free}} \frac{\|R(k) - C(k)H(k)\|_F^2}{\|R(k) - \hat{C}(k)H(k)\|_F^2} \ge 0\right)
$$
(10)

$$
= P\left(\left[\sum_{k,d_{free}} Tr\left\{\begin{array}{c} H^H(k)(C(k) - \hat{C}(k))^H \\ (C(k) - \hat{C}(k))H(k) \end{array}\right\}\right] - \beta \le 0\right)
$$
\n(11)

where $\beta = \sum_{k,dfree} \beta(k)$, and $\beta(k) = Tr\{H^H(k)(\hat{C}(k) - C(k))H_1K(k)(\hat{C}(k) - C(k))H_2K(k)(\hat{C}(k) - C(k))H_3K(k)(\hat{C}(k) - C(k))H_4K(k)(\hat{C}(k) - C(k))H_5K(k)(\hat{C}(k) - C(k))H_6K(k)(\hat{C}(k) - C(k))H_7K(k)(\hat{C}(k) - C(k))H_8K(k)(\hat{C}(k) - C(k))H_9K(k)(\hat{C}(k) - C(k))H_9K(k)(\hat{C}(k) - C$ $C(k)$ ^HN(k) + $N^{\overline{H}}(k)(\hat{C}(k) - C(k))H$. $\beta(k)$ s are zeromean, independent complex Gaussian random variables with variance $2N_0 ||(\hat{C}(k) - C(k))H||_F^2$. Then, β is
a zero-mean Gaussian random variable with variance a zero-mean Gaussian random variable with variance $2N_0 \sum_{k,d_{free}} ||(\hat{C}(k) - C(k))H||_F^2$. Note that, the upper bound in (10) is tight, since for high SNR values $C(k)$ = $C(k)$. Finally, PEP can be written as

$$
P(\underline{c} \to \underline{\hat{c}} | \mathbf{H}) \le P\left(\beta \ge \sum_{k, d_{free}} \|(C(k) - \hat{C}(k))H(k)\|_{F}^{2}\right)
$$

$$
\le Q\left(\sqrt{\frac{\sum_{k, d_{free}} \|(C(k) - \hat{C}(k))H(k)\|_{F}^{2}}{2N_{0}}}\right)
$$
(12)

where $Q(\cdot)$ is the well-known Q -function.

Let's define $D(k) = C(k) - \tilde{C}(k)$, which is still a $T \times N$ (generalized) complex orthogonal design (i.e., $D(k)$ satisfies (1)). $H(k)$ can be rewritten as

$$
H(k) = \mathbf{W}^{H}(k)\mathbf{h}
$$

$$
\mathbf{W}(k) = \begin{bmatrix} \frac{\mathbf{W}(k)}{\mathbf{0}_{L\times 1}} & \frac{\mathbf{0}_{L\times 1}}{\mathbf{W}(k)} & \cdots & \frac{\mathbf{0}_{L\times 1}}{\mathbf{0}_{L\times 1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\mathbf{0}_{L\times 1}}{\mathbf{0}_{L\times 1}} & \frac{\mathbf{0}_{L\times 1}}{\mathbf{0}_{L\times 1}} & \cdots & \frac{\mathbf{W}(k)}{\mathbf{W}(k)} \end{bmatrix}_{N L \times N}
$$

$$
\mathbf{h} = \begin{bmatrix} \frac{\mathbf{h}_{11}}{\mathbf{h}_{21}} & \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} & \cdots & \frac{\mathbf{h}_{1M}}{\mathbf{h}_{2M}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\mathbf{h}_{N1}}{\mathbf{h}_{N2}} & \cdots & \frac{\mathbf{h}_{NM}}{\mathbf{h}_{NN}} \end{bmatrix}_{N L \times M}
$$
(13)

where $\underline{0}_{L\times 1}$ denotes a zero vector of size $L \times 1$. Then,

$$
\sum_{k,d_{free}} \|D(k)H(k)\|_{F}^{2} = Tr{\mathbf{h}^{H} \mathbf{Z} \mathbf{h}}
$$

$$
\mathbf{Z} = \sum_{k,d_{free}} Z_{k}
$$

$$
Z_{k} = \mathbf{W}(k)D^{H}(k)D(k)\mathbf{W}^{H}(k) \qquad (14)
$$

From (1), $D^H(k)D(k) = |d(k)|^2 I_N$, where $|d(k)|^2$
 $|d_k(k)|^2 + |d_k(k)|^2 + |d_k(k)|^2$ is a non-zero posi- $\kappa(|d_1(k)|^2 + |d_2(k)|^2 + \ldots + |d_S(k)|^2)$ is a non-zero positive
constant with $d(k)$'s denoting the S complex numbers of constant with $d_i(k)$ s denoting the S complex numbers of $D(k)$, and I_N is the $N \times N$ identity matrix. Then, Z_k can be written as

$$
Z_{k} = \begin{bmatrix} A_{k} & 0_{L \times L} & \cdots & 0_{L \times L} \\ 0_{L \times L} & A_{k} & \cdots & 0_{L \times L} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{L \times L} & 0_{L \times L} & \cdots & A_{k} \end{bmatrix}_{N L \times NL}
$$

\n
$$
A_{k} = |d(k)|^{2} \begin{bmatrix} 1 & W_{K}^{k} & \cdots & W_{K}^{(L-1)k} \\ W_{K}^{-k} & 1 & \cdots & W_{K}^{(L-2)k} \\ \vdots & \vdots & \ddots & \vdots \\ W_{K}^{-(L-1)k} & W_{K}^{-(L-2)k} & \cdots & 1 \end{bmatrix}_{L \times L}
$$

\n(15)

where each A_k is also Hermitian with a square root $\underline{W}(k)d^*(k)$ such that $A_k = W(k)d^*(k)(W(k)d^*(k))^H$, and $rank(A_k) =$ 1. Then, $rank(Z_k) = N$. It is shown in [5], the rank of the $L \times L$ matrix $\mathbf{A} = \sum_{k,d_{free}} A_k$ is $\min(d_{free}, L)$. Then,

$$
\mathbf{Z} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{L \times L} & \cdots & \mathbf{0}_{L \times L} \\ \mathbf{0}_{L \times L} & \mathbf{A} & \cdots & \mathbf{0}_{L \times L} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{L \times L} & \mathbf{0}_{L \times L} & \cdots & \mathbf{A} \end{bmatrix}_{N L \times NL}
$$
(17)

has rank $r = N \min(d_{free}, L)$. From linear algebra, it is known that any matrix with a square root is positive

semidefinite [6], [20]. Also, any non-negative linear combination of positive semidefinite matrices is positive semidefinite. Therefore A_k s and A are positive semidefinite, and similarly Z_k s (with a square root $\mathbf{W}(k)D^H(k)$) and **Z** are positive semidefinite. Then, the singular value decomposition of **Z** can be written as [20]

$$
\mathbf{Z} = V^H \Lambda V \tag{18}
$$

where V is a unitary matrix and Λ is a diagonal matrix with eigenvalues of **Z**, $\{\lambda_i\}_{i=1}^{NL}$, on the diagonal. Note that the eigenvalues of the positive semidefinite matrix **Z** are real and non-negative. As a result,

$$
\sum_{k,d_{free}} ||D(k)H(k)||_F^2 = Tr{\mathbf{h}^H \mathbf{Z} \mathbf{h}} = Tr{\mathbf{h}^H V^H \Lambda V \mathbf{h}}
$$

$$
= \sum_{m=1}^{M} \sum_{n=1}^{NL} \lambda_n |v_{nm}|^2 \tag{19}
$$

where v_{nm} , $n = 1, \ldots, NL$, $m = 1, \ldots, M$ are the elements of the $NL \times M$ matrix Vh. Note that each v_{nm} is a complex Gaussian random variable. Then, $|v_{nm}|$ are Rayleigh distributed with pdf $2|v_{nm}|e^{-|v_{nm}|^2}$. Using an upper bound
for the O function $O(x) < (1/2)e^{-x^2/2}$. PEP can be found as for the Q function $Q(x) \le (1/2)e^{-x^2/2}$, PEP can be found as

$$
P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}) = E\left[P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}|\mathbf{H})\right]
$$

\n
$$
\leq E\left[\frac{1}{2}\exp\left(-\frac{\sum_{m=1}^{M} \sum_{n=1}^{NL} \lambda_n |v_{nm}|^2}{4N_0}\right)\right] = \frac{1}{\prod_{n=1}^{NL} \left(1 + \frac{\lambda_n}{4N_0}\right)^M}
$$
(20)

For $rank(\mathbf{Z}) = r = N min(d_{free}, L)$, without loss of generality we can order the λ_n 's such that, $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_r$ and $\lambda_{r+1} = \ldots = \lambda_{NL} = 0$. Using $N_0 = N/SNR$ from Section IV, PEP becomes upper bounded by

$$
P(\underline{c} \to \hat{\underline{c}}) \le \frac{1}{\prod_{n=1}^{r} (1 + \frac{\lambda_n SNR}{4N})^M}
$$

$$
\simeq \left(\prod_{l=1}^{r} \lambda_l\right)^{-M} \left(\frac{SNR}{4N}\right)^{-rM} \quad \text{for high } SNR
$$
 (21)

It is clearly evident from (21) that the BICM-STBC-OFDM system successfully reaches to the diversity order of $NM \min(d_{free}, L).$

VI. SIMULATION RESULTS

In our simulations, each OFDM symbol has 64 tones, and has a duration of 4 μ s of which 0.8 μ s is CP. 250 bytes are sent with each packet and the channel is assumed to be the same through the transmission of one packet. The maximum delay spread of the channel is set to be ten times the root mean square (rms) delay spread. The system has two transmit antennas for all the results presented in this section. For BICM-STBC-OFDM, Alamouti's code [7] is used in order to implement two transmit antennas.

Figure 3 shows the results for the industry standard (133,171) 1/2 rate 64 states $d_{free} = 10$ convolutional code with different rms delay spread values. It can be seen from the figures that as the number of taps increases in the channel, the diversity order of BICM-STBC-OFDM increases up to the maximum diversity of $NM \min(d_{free}, L)$. Note that, as the number of receive antennas is increased, the diversity order gets multiplied in the figures. For 2 transmit 4 receive antenna case, even at low SNR values, the performance curve is extremely steep.

Fig. 3. BICM-STBC-OFDM results using $1/2$ rate 64 states $d_{free} = 10$ code

Figure 4 shows the performance curves for 4 state BICM-STBC-OFDM, 4 state QPSK SOSTTC [10] with OFDM, and 4 State QPSK STTC [6] with OFDM. 4 state 1/2 rate $d_{free} = 5$ convolutional code [21] with 16 QAM modulation is used for BICM-STBC-OFDM so that all the systems transmit 2 bits/sec/Hz at each tone. As can be seen from the figures, BICM-STBC-OFDM reaches a higher diversity value for frequency selective channels.

Fig. 4. Comparison between BICM-STBC-OFDM, SOSTTC-OFDM and STTC-OFDM

VII. CONCLUSION

Diversity order being defined as the negative slope of the error rate vs signal to noise ratio curve, is a dominant criterion for the performance of wireless communication systems. In this paper we introduced BICM-STBC-OFDM in order to exploit diversity in space and frequency. We have shown both analytically and via simulations that BICM-STBC-OFDM reaches the maximum diversity order in space and frequency by using an appropriate convolutional code. If the convolutional code being used has a minimum Hamming distance of d_{free} , we showed that the diversity order of BICM-STBC-OFDM is $NM \min(d_{free}, L)$ for a system with N transmit and M receive antennas over an L tap frequency selective fading channel.

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