Performance Analysis of Beamforming for MIMO OFDM with BICM

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***Abstract*— In this paper we will show and quantify both analytically and via simulations that the use of channel knowledge at the transmitter, the technique known as beamforming, achieves the maximum diversity in space when the best eigenmode is used (single beamforming). Furthermore, we will investigate beamforming in conjunction with next generation wireless local area networks (WLANs). It is known that the widely used technique in WLANs, bit interleaved coded modulation (BICM) with orthogonal frequency division multiplexing (OFDM), can achieve the maximum frequency diversity order that is inherited in the channel. We will show that the combination of BICM, single beamforming, and OFDM also leads to the maximum diversity order in space and frequency domains. In other words, for systems with N transmit and M receive antennas, BICM - Beamforming - OFDM (BBO) can achieve a diversity order of NML over L-tap frequency selective channels by using an appropriate convolutional code. In addition to having a substantial diversity order, simulation results show that beamforming and BBO introduce significant coding gains when compared to other systems based on space time block codes (STBC) with the same diversity order.**

I. Introduction

In order to combat the severe conditions of wireless channels, wireless systems should achieve a high diversity order. Many diversity techniques, some of which are temporal, frequency, code, and spatial have been developed.

In recent years deploying multiple transmit and receive antennas has become an important tool to improve diversity. Multi-input multi-output (MIMO) systems allow significant diversity gains for wireless communications. MIMO wireless systems incorporating diversity generally can be grouped into two. The first group requires the channel state information (CSI) at the receiver, but not at the transmitter. These systems include space-time (ST) codes and some important results can be listed as [1], [2], and the references therein.

The second group requires CSI both at the transmitter and the receiver ends. This approach is known as beamforming. Beamforming separates the MIMO channel into parallel independent subchannels. When the best subchannel is used, the technique is called single beamforming [3]. If more than one subchannel is used, the technique is called multiple beamforming [3]. In this paper we focus on single beamforming, and from now we will refer to it simply as beamforming for simplicity. A brief overview and the performance analysis of beamforming are presented in Section II. It is shown that beamforming can achieve the maximum spatial diversity order and a tight bound for error probability is given.

In this paper we also investigate beamforming in the context of next generation WLANs. It is known that the widely used technique in WLANs, bit interleaved coded modulation (BICM) with orthogonal frequency division multiplexing (OFDM), can achieve the maximum frequency diversity order that is inherited in the channel [4]. In Section III we present a system as a combination of BICM ([5]), beamforming, and OFDM, hence named it as BBO. Our analysis shows that BBO can achieve the maximum diversity order in space and frequency by using an appropriate convolutional code. In other words, for systems with N transmit and M receive antennas, BBO achieves a diversity order of NML over L-tap frequency selective channels.

Simulation results supporting our analysis are given in Section IV. Besides the simulation results based on the channel model of Section III, we present simulation results using the IEEE MIMO wireless channel models [6], [7], [8]. Finally, we end the paper with a brief conclusion in Section V.

II. Beamforming

A. Overview

If CSI is available both at the transmitter and the receiver, MIMO systems can benefit from significant diversity and coding gains by using beamforming. CSI can be obtained at the transmitter by using a feedback channel. One restriction is that the delay introduced by the feedback channel must be shorter than the coherence time of the wireless channel.

In this paper, however, we assume that the perfect CSI is available both at the transmitter and at the receiver. In such a case, the beamforming vectors are obtained via the singular value decomposition (SVD) of the channel. Let's denote the quasi-static Rayleigh flat fading $N \times M$ MIMO channel as H. Then, the SVD of H can be written as

$$H = U\Lambda V^H = [u_1 u_2 \cdots u_N]\Lambda \left[\; v_1 v_2 \ldots v_M \;\right]^H \qquad (1)$$

where Λ is a $N \times M$ matrix with singular values, $\{\lambda_i\}_{i=1}^{min(N,M)}$, in decreasing order on the main diagonal. U and V are two unitary matrices of size $N \times N$ and $M \times M$, respectively.

The beamforming vectors to be used at the transmitter and the receiver sides are the first columns of U and V, respectively, corresponding to the largest singular value of H. Then, the overall system can be represented by

$$y = xu_1^H H v_1 + \eta v_1 = \lambda_1 x + n \tag{2}$$

where λ_1 is the largest singular value of H, x is the transmitted symbol, and η is complex additive white Gaussian noise (AWGN) vector of size $1 \times M$ with zero mean and variance $N_0 = 1/SNR$. The elements of H are modeled as complex Gaussian random variables with zero mean and 0.5 variance per complex dimension. Note that, the average total transmit power at the transmitter is assumed to be 1. Therefore, the received signal-to-noise ratio is SNR with the given channel and noise models.

B. Performance Analysis

In this section, by analyzing the pairwise error probability (PEP), we will show that beamforming achieves the diversity order of NM for arbitrary N and M. References [9] and [10] emphasize the same result but do not give a formal proof for an arbitrary (N, M) pair.

Assume that the symbol x is sent and $\hat{x}$ is detected. Then, using the maximum likelihood (ML) criterion, the PEP of x and $\hat{x}$ given CSI can be written as

$$\begin{aligned} P(x \to \hat{x}|\mathbf{H}) &= P\left(|y - \lambda_1 x|^2 \geq |y - \lambda_1 \hat{x}|^2\right) \\ &= P\left(\beta - |\lambda_1|^2|x - \hat{x}|^2 \geq 0\right) \leq Q\left(\sqrt{\frac{|\lambda_1|^2 d_{min}^2}{2N_0}}\right) \end{aligned} \tag{3}$$

where $\beta = \lambda_1(\hat{x} - x)n^* + {\lambda_1}^*(\hat{x} - x)^* n$, d_{min} is the minimum Euclidean distance in the constellation, and $Q(\cdot)$ is the well known Q-function. For given H, β is an independent zero-mean complex Gaussian random variable with variance $2N_0|\lambda_1|^2|x - \hat{x}|^2$. Using an upper bound for the Q function $Q(x) \leq (1/2)e^{-x^2/2}$, PEP can be found as

$$P(x \to \hat{x}) = E\left[P(x \to \hat{x}|\mathbf{H})\right] \leq E\left[\frac{1}{2}e^{-\frac{|\lambda_1|^2 d_{min}^2}{4N_0}}\right]. \tag{4}$$

Without loss of generality, we assume $N \leq M$. Since λ_1 is the maximum singular value, then

$$|\lambda_1|^2 \geq \frac{|\lambda_1|^2 + |\lambda_2|^2 + \cdots + |\lambda_N|^2}{N} . \tag{5}$$

PEP can given by

$$P(x \to \hat{x}) \leq E\left[\frac{1}{2}\exp\left(-\frac{\theta d_{min}^2}{4N_0 N}\right)\right] \tag{6}$$

where

$$\theta = \sum_{i=1}^{N} |\lambda_i|^2 = \|\mathbf{H}\|_F^2 = \sum_{i,j} |h_{ij}|^2 \tag{7}$$

and $\|(\cdot)\|_F^2$ is the Frobenius norm. Note that, since θ is the summation of NM magnitude squared complex Gaussian random variables, h_{ij}'s, θ is a chi-square random variable with $2NM$ degrees of freedom. Taking expectation with respect to the chi-square probability density function [11], PEP can be calculated as

$$P(x \to \hat{x}) \leq \frac{1}{2^{NM+1}}\left(\frac{d_{min}^2}{4N_o N} + \frac{1}{2}\right)^{-NM} \tag{8}$$

At high SNR, since $N_0 = 1/SNR$, PEP is simplified to

$$P(x \to \hat{x}) \leq \frac{1}{2^{NM+1}}\left(\frac{d_{min}^2}{4N}SNR\right)^{-NM}. \tag{9}$$

From (9), it is easy to see that the diversity order of beamforming is NM. The diversity analysis of multiple beamforming is given in [12].

III. BICM Beamforming OFDM (BBO)

A. System Model

The system consists of N transmit and M receive antennas; and one OFDM symbol has K subcarriers. A convolutional encoder is used to code the bits. The output bits of the encoder are interleaved within one OFDM symbol to avoid extra delay requirement to start decoding at the receiver. After interleaving, the output bit $c_{k'}$ is mapped onto the tone $x(k)$ at the ith bit location using the signal map χ. It is assumed that an appropriate length of cyclic prefix (CP) is used for each OFDM symbol. By doing so, OFDM converts the frequency selective channel into parallel flat fading channels, denoted as $H(k)$ for $k = 1, 2, \ldots, K$. Each symbol for each tone is then multiplied by the corresponding vector $u_1^H(k)$ (see Section II) of the flat MIMO channel that is seen by each subcarrier. Following beamforming, OFDM is applied at each transmit antenna. The (n, m)th component of the $N \times M$ MIMO channel $H(k)$ is given by

$$\begin{aligned} &H_{nm}(k) = \underline{\mathrm{W}}^H(k)\underline{\mathrm{h}}_{nm} \\ &\underline{\mathrm{W}}(k) = [1 \quad W_K^k \quad \cdots \quad W_K^{(L-1)k}]^H, \text{ where } W_K^k \triangleq e^{-i\frac{2\pi}{K}} \\ &\underline{\mathrm{h}}_{nm} = [h_{nm}(0) \quad h_{nm}(1) \quad \cdots \quad h_{nm}(L-1)]^T \end{aligned} \tag{10}$$

where $\underline{\mathrm{h}}_{nm}$ represents the L tap frequency selective channel from the transmit antenna n to the receive antenna m. Each tap is assumed to be statistically independent and modeled as a zero mean complex Gaussian random variable with variance $1/L$. It is assumed that the taps of the frequency selective channel are located at the integer multiples of subcarrier symbol duration. The fading model is assumed to be quasi-static, i.e., the fading coefficients are constant over the transmission of one packet, but independent from one packet transmission to the next.

When the beamforming of Section II is applied on each subcarrier, the received signal is given by

$$y(k) = x(k)\lambda_1(k) + n(k) \tag{11}$$

where $n(k) = N(k)v_1(k)$, and $N(k)$ is complex additive white Gaussian noise of size $1 \times M$ with zero mean and variance $N_0 = 1/SNR$. Note that, the average total power transmitted from all the transmit antennas at each subcarrier is scaled to 1. Therefore, with the given channel and noise models the received signal to noise ratio at each receive antenna is SNR.

B. Performance Analysis

In the following section we will show, by calculating the PEP that BBO can achieve the maximum diversity order in frequency and space. Note that, it was shown in [13] and [14] that the maximum achievable diversity order in MIMO frequency selective fading channels is NML. Let's denote the minimum Hamming distance of the convolutional code used for BICM as d_{free}. It is assumed in this section that $d_{free} \leq L$, and it will be shown that BBO achieves the diversity order of NMd_{free}. Also, as will be presented in Section IV if $d_{free} > L$, BBO achieves the maximum frequency diversity order of NML. Assume the code sequence $\underline{c}$ is transmitted and $\underline{\hat{c}}$ is detected. Then, using the bit metrics given in [5] and (11), the PEP of $\underline{c}$ and equation $\underline{\hat{c}}$ given CSI can be written as

$$P(\underline{c} \rightarrow \underline{\hat{c}}|\mathbf{H}) = P\left(\sum_{k'} \min_{x \in \chi^i_{c_{k'}}} |y(k) - x\lambda_1(k)|^2 \geq \sum_{k'} \min_{x \in \chi^i_{\hat{c}_{k'}}} |y(k) - x\lambda_1(k)|^2\right). \quad (12)$$

where χ^i_b is a subconstellation of χ, where the bit $b \in \{0, 1\}$ is on the ith location of the symbol map.

For a convolutional code with rate k_0/n_0, and minimum Hamming distance d_{free}, the Hamming distance between $\underline{c}$ and $\underline{\hat{c}}$, $d(\underline{c} - \underline{\hat{c}})$, is at least d_{free}. Assume $d(\underline{c} - \underline{\hat{c}}) = d_{free}$ for $\underline{c}$ and $\underline{\hat{c}}$ under consideration for PEP analysis, which is the worst case scenario between any two codewords. Then, $\chi^i_{c_{k'}}$ and $\chi^i_{\hat{c}_{k'}}$ are equal to one another for all k' except for d_{free} distinct values of k'. Therefore, the inequality on the right hand side of (12) shares the same terms on all but d_{free} summation points. Hence, the summations can be simplified to only d_{free} terms for PEP analysis.

$$P(\underline{c} \rightarrow \underline{\hat{c}}|\mathbf{H}) = P\left(\sum_{k', d_{free}} \min_{x \in \chi^i_{c_{k'}}} |y(k) - x\lambda_1(k)|^2 \geq \sum_{k', d_{free}} \min_{x \in \chi^i_{\hat{c}_{k'}}} |y(k) - x\lambda_1(k)|^2\right) \quad (13)$$

where $\sum_{k', d_{free}}$ denotes that the summation is taken with index k' over d_{free} different values of k'.

Note that for binary codes and for the d_{free} points at hand, $\hat{c}_{k'} = \bar{c}_{k'}$, where $\bar{(\cdot)}$ denotes the binary complement of $(\cdot)$. For the d_{free} bits let's denote

$$\tilde{x}(k) = \arg \min_{x \in \chi^i_{c_{k'}}} |y(k) - x\lambda_1(k)|^2$$
$$\hat{x}(k) = \arg \min_{x \in \chi^i_{\bar{c}_{k'}}} |y(k) - x\lambda_1(k)|^2. \quad (14)$$

It is easy to see that $\tilde{x}(k) \neq \hat{x}(k)$ since $\tilde{x}(k) \in \chi^i_{c_{k'}}$ and $\hat{x}(k) \in \chi^i_{\bar{c}_{k'}}$ where $\chi^i_{c_{k'}}$ and $\chi^i_{\bar{c}_{k'}}$ are complement sets of constellation points within the signal constellation set χ. Also, $|y(k) - x(k)\lambda_1(k)|^2 \geq |y(k) - \tilde{x}(k)\lambda_1(k)|^2$ and $x(k) \in \chi^i_{c_{k'}}$.

For convolutional codes, due to their trellis structure, d_{free} distinct bits between any two codewords occur in consecutive trellis branches. The bit interleaver can be designed such that the consecutive bits on the trellis branches that d_{free} bits span are mapped onto distinct symbols. This guarantees that there exist d_{free} distinct pairs of $(\tilde{x}(k), \hat{x}(k))$, and d_{free} distinct pairs of $(x(k), \hat{x}(k))$. The PEP of BBO can be rewritten as

$$\begin{aligned} P(\underline{c} \rightarrow \underline{\hat{c}}|\mathbf{H}) &= P\Big(\sum_{k, d_{free}} |y(k) - \tilde{x}(k)\lambda_1(k)|^2 - |y(k) - \hat{x}(k)\lambda_1(k)|^2 \geq 0\Big) \\ &\leq P\left(\beta \geq \sum_{k, d_{free}} |\left(x(k) - \hat{x}(k)\right)\lambda_1(k)|^2\right) \\ &\leq Q\left(\sqrt{\frac{\sum_{k, d_{free}} d^2_{min}|\lambda_1(k)|^2}{2N_0}}\right) \end{aligned} \quad (15)$$

where $\beta = \sum_{k, d_{free}} \beta(k)$ and $\beta(k) = (\hat{x}(k) - x(k))^* \lambda_1^*(k) n(k) + (\hat{x}(k) - x(k))\lambda_1(k) n^*(k)$. For given $\mathbf{H}$, $\beta(k)$s are independent zero-mean complex Gaussian random variables with variance $2N_0|(\hat{x}(k) - x(k))\lambda_1(k)|^2$. Consequently, β is a complex Gaussian random variable with zero mean and variance $2N_0 \sum_{k, d_{free}} |(\hat{x}(k) - x(k))\lambda_1(k)|^2$. Using an upper bound for the Q function $Q(x) \leq (1/2)e^{-x^2/2}$, PEP can be upper bounded as

$$\begin{aligned} P(\underline{c} \rightarrow \underline{\hat{c}}) &= E\left[P(\underline{c} \rightarrow \underline{\hat{c}}|\mathbf{H})\right] \\ &\leq E\left[\frac{1}{2}\exp\left(-\frac{\sum_{k, d_{free}} d^2_{min}|\lambda_1(k)|^2}{4N_0}\right)\right]. \end{aligned} \quad (16)$$

OFDM converts the frequency selective channel into parallel, independent flat fading channels. Therefore, the summation in the exponential of (16) can be moved to outside of the expectation. Then, using (4) through (9) for each subcarrier

$$\begin{aligned} P(\underline{c} \rightarrow \underline{\hat{c}}) &\leq \prod_{k, d_{free}} E\left[\frac{1}{2}\exp\left(-\frac{d^2_{min}|\lambda_1(k)|^2}{4N_0}\right)\right] \\ &\leq 2^{-(NM+1)d_{free}}\left(\frac{d^2_{min}}{4N}SNR\right)^{-NMd_{free}} \end{aligned} \quad (17)$$

As can be seen from (17), the diversity order of BBO is NMd_{free} for $d_{free} \leq L$. Simulation results also showed that (see Section IV) if $L < d_{free}$ then BBO achieves the maximum diversity order of NML in space and frequency. This is a significant result since, for example, the industry standard $1/2$ rate 64 state (133,171) convolutional encoder has $d_{free} = 10$. Therefore, a BBO system with 4 transmit and 4 receive antennas can achieve a diversity order of 160, which

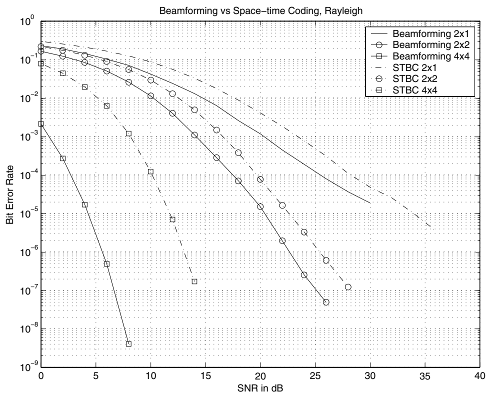

Fig. 1. Beamforming vs STBC results for flat fading channels

leads to very steep curves in error rate vs signal to noise ratio figures.

It is given in [15] and [16] that a very low complexity decoder for BICM-OFDM can be implemented. The same decoder can be used for BBO as well: Instead of using the single-input single-output (SISO) channel value of BICM-OFDM for the decoder ([15] and [16]), one should use $\lambda_1(k)$. Hence, BBO provides a very high performance and easy to decode system.

IV. Simulation Results

A. Beamforming Results

In order to illustrate the validity of the theoretical results in Section II, simulations were carried out for 2×1, 2×2 and 4×4 systems. For 4×4 simulations, QPSK is used for beamforming and 16 QAM is used for half rate STBC ([2]) in order to achieve the same data rate. Performance of both beamforming and STBC systems are compared. As shown in Figure 1, both beamforming and STBC achieve diversity order of 2, 4, and 16 for 2×1, 2×2, and 4×4 scenarios, respectively. In addition, beamforming has an 2.5 dB coding gain when compared to the Alamouti code. This gain is achieved by the SNR gain at the receiver due the largest singular value of the channel. As the number of antennas increases, the mean value of the largest eigenvalue of the channel increases. Therefore, the coding gain is higher for larger number of antennas, almost 7 dB for the 4×4 case.

B. BBO Results

The BBO system used in the simulations has 64 subcarriers for one OFDM symbol. Each OFDM symbol has a duration of $4\mu s$ of which $0.8\mu s$ is CP. 250 bytes are sent in each packet.

1) Results using the Channel Model of Section III: The channel is modeled as given in Section III-A. The maximum delay spread of the frequency selective channel is set as 10 times the root mean square (rms) delay spread. The delay spread of the channel is given as the rms value in the figures below.

Figure 2 illustrates the results of BBO as compared to a system as a combination of BICM, STBC, and OFDM ([17]). Both systems use the same convolutional code with 4 states and $d_{free} = 5$. The best convolutional code for 4 states is picked from the tables of [18]. The output bits of the encoder are interleaved using the interleaver of IEEE 802.11a [19] within one OFDM symbol. The interleaved bits are then modulated using 16 QAM. Both systems deploy 2 transmit and 1 receive antennas. The Alamouti code is used to implement BICM-STBC-OFDM. As can be seen from the figures, as the delay spread of the channel increases the diversity order of BBO increases as well. Once the delay spread reaches the value of d_{free}, the diversity order does not increase. Error rate vs signal to noise ratio curves of BBO over 50 ns and 25 ns rms delay spread channels go parallel together. However, there is a slight coding gain for 50 ns channel. The figures also illustrate that BBO introduces a coding gain when compared to BICM-STBC-OFDM. Although, both systems succeed in achieving the same diversity order under the same conditions, BBO shows about 2-3 dB improvement for the 2 transmit and 1 receive antennas case.

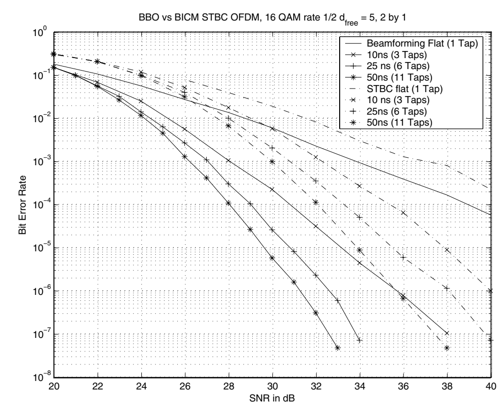

Fig. 2. BBO vs BICM STBC OFDM over different frequency selective channels. 2 transmit 1 receive antennas, 1/2 rate, 4 states $d_{free} = 5$.

Figure 3 presents the simulation results for BBO as compared to BICM-STBC-OFDM when both systems use the industry standard 1/2 rate (133,171) 64 state $d_{free} = 10$ convolutional code. The diversity order of BBO increases with the increasing delay spread of the channel. It can be seen that with increasing transmit and receive antennas, the diversity order of BBO increases as well. As mentioned earlier, BBO introduces 2-3 dB coding gain when compared to BICM-STBC-OFDM for two transmit antennas case. Simulation results for 4 transmit and 4 receive antennas case are presented in Figure 3. Note that, since there is no full rate STBC for 4 antennas, we used 1/2 rate STBC [2]. When BBO has the same rate as BICM-STBC-OFDM by using QPSK instead of 16 QAM for 4 transmit antennas, the coding gain is about 6 dB.

2) Results using the IEEE MIMO Channel Models: In this section we present simulation results using the channel models of IEEE 802.11n workgroup [6], [7], [8]. Channel models are created with increasing delay spreads. Model B has 9 taps with power decay profile given in [6], whereas Model D and

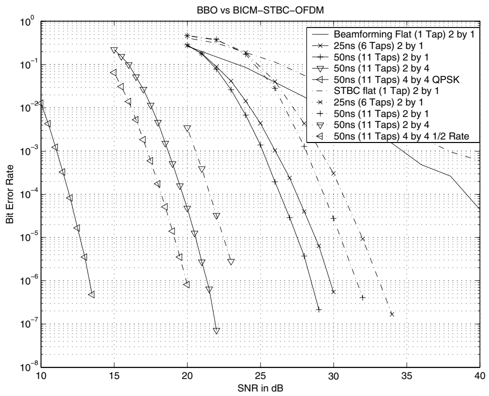

Fig. 3. BBO vs BICM STBC OFDM over different frequency selective channels. 1/2 rate, 64 states, $d_{free} = 10$.

E have 18 taps. As can be seen in Figure 4, the diversity order of BBO increases with the increasing delay spread. Note that, once the diversity order NMd_{free} is achieved, the slope of the curves do not increase with delay spread (Model D and Model E results are close to one another). BBO shows 2-3 dB coding gain for 2 transmit antennas case.

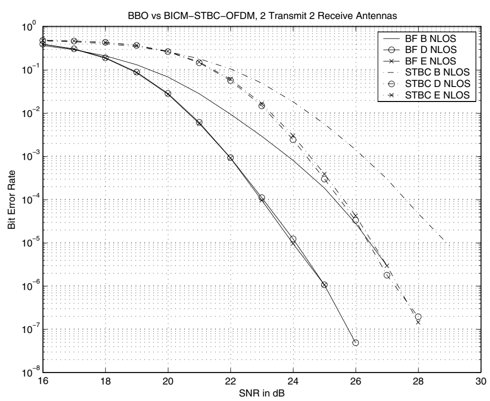

Fig. 4. BBO vs BICM STBC OFDM over IEEE MIMO channel models. 1/2 rate, 64 states, $d_{free} = 10$.

V. Conclusion

In this paper we first formally showed that single beamforming can achieve the maximum spatial diversity order. Simulation results showed significant coding gains for beamforming as compared to STBC where the channel state information is not required at the transmitter.

We investigated beamforming in the context of high speed broadband wireless communications. In this context, we combined beamforming with BICM-OFDM (which is widely used in existing WLANs). We called the resulting system BBO. We provided an analysis showing that BBO can achieve the maximum diversity order in space and frequency by using an appropriate convolutional code. In other words, for N transmit and M receive antennas, BBO can achieve a diversity order of NML for L-tap frequency selective channels.

In addition to having a substantial diversity order, simulation results showed that BBO introduces significant coding gains when compared to other systems based on STBC with the same diversity order. As the number of transmit antennas increases, the coding gains increase as well, since there is no full-rate complex STBC for more than 2 antennas.

A way to implement a low complexity decoding for BBO is also described. As a result, BBO provides a very high performance (maximum diversity in space and frequency, and high coding gain), and easy to decode system for broadband wireless communications, provided CSI is available both at the tramsitter and receiver.

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