Achieving Full Spatial Multiplexing and Full Diversity in Wireless Communications

Enis Akay, Ersin Sengul, and Ender Ayanoglu

Center for Pervasive Communications and Computing
Department of Electrical Engineering and Computer Science
The Henry Samueli School of Engineering
University of California, Irvine
Irvine, California 92697-2625
Phone: +1 949 824 9797
Email: eakay@uci.edu, esengul@uci.edu, ayanoglu@uci.edu

***Abstract*— It is well-known that using multiple antennas provides a substantial capacity and diversity increase for wireless communication systems. A multi-input multi-output (MIMO) technique that utilizes the channel knowledge both at the transmitter and the receiver is known as beamforming. Beamforming separates a MIMO channel into parallel subchannels. It was previously shown that uncoded beamforming achieves a diversity order of $(N-S+1)(M-S+1)$ if S symbols are transmitted simultaneously for N transmit and M receive antennas. Hence, there is a significant drop in the diversity order (and performance) of the system with increased spatial multiplexing. In this paper, we introduce bit interleaved coded multiple beamforming and name the system BICMB. We provide interleaver design criteria such that the resulting system achieves full spatial multiplexing of $\min(N,M)$ and full spatial diversity of NM. Simulation results show that BICMB, due to its ability of maintaining the maximum diversity order even at full spatial multiplexing, provides substantial performance gain when compared to the best spatial multiplexing systems.**

I. Introduction

Achieving high throughput while maintaining robustness is one of the biggest challenges in wireless communications. In general, practical systems sacrifice the crucial diversity that is required for high performance in exchange of higher data rates.

It is known that multi-input multi-output (MIMO) systems provide significant capacity increase [1]. MIMO systems also achieve a high diversity order. Some of the high diversity order achieving systems do not require channel state information (CSI) at the transmitter (e.g., space-time codes [2]). A technique that provides high diversity and coding gain with the help of CSI at the transmitter is known as beamforming. Beamforming separates the MIMO channel into parallel subchannels. Therefore, multiple streams of data can be transmitted easily. Single beamforming (i.e., sending one symbol at a time) was shown to achieve the maximum diversity in space with a substantial coding gain compared to space time codes [3]. If more than one symbol at a time are transmitted, then the technique is called multiple beamforming. For uncoded multiple beamforming systems, it was shown that while the data rate increases, one loses the diversity order with the increasing number of streams used over flat fading channels [4].

Bit interleaved coded modulation (BICM) was introduced as a way to increase the code diversity [5], [6]. BICM has been deployed with OFDM, and MIMO OFDM systems to achieve high diversity orders [7], [8], [9], [10], [11] (and references therein). In Section II, we introduce bit interleaved coded multiple beamforming. We show that with the inclusion of BICM to the system, one does not lose the diversity order with multiple beamforming even when all the subchannels are used. That is, in Section III we show that BICMB achieves full diversity NM, and full spatial multiplexing $\min(N,M)$ for a system with N transmit and M receive antennas. In this paper, spatial multiplexing is defined as the number of symbols transmitted simultaneously over N transmit antennas. In order to guarantee full diversity, we provide design criteria for the interleaver.

We provide simulation results in Section IV supporting our analytical analysis. Finally, we end the paper with a brief conclusion in Section V.

Notation: N is the number of transmit antennas, M is the number of receive antennas. The minimum hamming distance of a convolutional code is defined as d_{free}. S denotes the total number of symbols transmitted at a time, in other words the total number of streams used. The minimum distance between two constellation points is given by d_{min}. The superscripts $(\cdot)^H$, $(\cdot)^*$, and $\bar{(\cdot)}$ denote the Hermitian, complex conjugate, and binary complement, respectively.

II. Bit Interleaved Coded Multiple Beamforming (BICMB): System Model

BICMB is a combination of BICM and multiple beamforming. Output bits of a binary convolutional encoder are interleaved and then mapped over a signal set $\chi \subseteq \mathbb{C}$ of size $|\chi| = 2^m$ with a binary labeling map $\mu : \{0,1\}^m \rightarrow \chi$. The d_{free} of the convolutional encoder should satisfy $d_{free} \geq S$. The interleaver is designed such that the consecutive coded bits are

1) mapped over different symbols,

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2) transmitted over different subchannels that are created by beamforming.

The reasons for the interleaver design are given in Section III. Gray encoding is used to map the bits onto symbols. During transmission, the code sequence $\underline{c}$ is interleaved by π, and then mapped onto the signal sequence $\underline{x} \in \chi$.

Beamforming separates the MIMO channel into independent subchannels. The beamforming vectors used at the transmitter and the receiver can be obtained by the singular value decomposition (SVD) [12] of the MIMO channel. Let H denote the quasi-static, flat fading $N \times M$ MIMO channel. Then the SVD of H can be written as

$$H = U\Lambda V^H = [u_1\, u_2 \dots u_N]\Lambda[v_1\, v_2 \dots v_M]^H \tag{1}$$

where U and V are $N \times N$ and $M \times M$ unitary matrices, respectively, and Λ is a $N \times M$ diagonal matrix with singular values of H, λ_i, on the main diagonal with decreasing order. If S symbols are transmitted at the same time, then the system input-output relation at the k^{th} time instant can be written as

$$\underline{y}_k = \underline{x}_k[u_1\, u_2 \dots u_S]^H H[v_1\, v_2 \dots v_S] + \underline{n}_k[v_1\, v_2 \dots v_S] \tag{2}$$

$$y_{k,s} = \lambda_s x_{k,s} + n_{k,s}, \text{ for } s = 1, 2, \dots, S \tag{3}$$

where $\underline{n}_k$ is $1 \times M$ additive white Gaussian noise with zero-mean and variance $N_0 = N/SNR$. Note that, the total power transmitted is scaled as N. The channel elements h_{nm} are modeled as zero-mean, unit-variance complex Gaussian random variables. Consequently, the received signal-to-noise ratio is SNR.

For an $N \times M$ uncoded multiple beamforming system, if S symbols are transmitted at a time, then it was shown that the diversity order for the uncoded multiple beamforming is equal to $(N - S + 1)(M - S + 1)$ [4].

The bit interleaver of BICMB can be modeled as $\pi : k' \to (k, s, i)$ where k' denotes the original ordering of the coded bits $c_{k'}$, k denotes the time ordering of the signals $x_{k,s}$ transmitted, s denotes the subchannel used to transmit $x_{k,s}$, and i indicates the position of the bit $c_{k'}$ on the symbol $x_{k,s}$.

Let χ_b^i denote the subset of all signals $x \in \chi$ whose label has the value $b \in \{0, 1\}$ in position i. Then, the ML bit metrics can be given by using (3), [5], [6]

$$\gamma^i(y_{k,s}, c_{k'}) = \min_{x \in \chi^i_{c_{k'}}} |y_{k,s} - x\lambda_s|^2. \tag{4}$$

The ML decoder at the receiver can make decisions according to the rule

$$\hat{\underline{c}} = \arg\min_{\underline{c} \in \mathcal{C}} \sum_{k'} \gamma^i(y_{k,s}, c_{k'}). \tag{5}$$

III. BICMB: Pairwise Error Probability Analysis

In this section we are going to show that by using BICM, and the given interleaver design criteria, coded multiple beamforming can achieve full spatial diversity order of NM while transmitting $S \le \min(N, M)$ symbols at a time. Assume the code sequence $\underline{c}$ is transmitted and $\hat{\underline{c}}$ is detected. Then, using (4), the PEP of $\underline{c}$ and $\hat{\underline{c}}$ given CSI can be written as

$$P(\underline{c} \to \hat{\underline{c}}|H) = P\left(\sum_{k'} \min_{x \in \chi^i_{c_{k'}}} |y_{k,s} - x\lambda_s|^2 \ge \sum_{k'} \min_{x \in \chi^i_{\hat{c}_{k'}}} |y_{k,s} - x\lambda_s|^2\right) \tag{6}$$

where $s \in \{1, 2, \dots, S\}$.

For a convolutional code with rate k_0/n_0, the Hamming distance between $\underline{c}$ and $\hat{\underline{c}}$, $d(\underline{c} - \hat{\underline{c}})$, is at least d_{free}. Assume $d(\underline{c} - \hat{\underline{c}}) = d_{free}$ for $\underline{c}$ and $\hat{\underline{c}}$ under consideration for PEP analysis. Then, $\chi^i_{c_{k'}}$ and $\chi^i_{\hat{c}_{k'}}$ are equal to one another for all k' except for d_{free} distinct values of k'. Therefore, the inequality on the right hand side of (6) shares the same terms on all but d_{free} summation points. Hence, the summations can be simplified to only d_{free} terms for PEP analysis.

$$P(\underline{c} \to \hat{\underline{c}}|H) = P\left(\sum_{k', d_{free}} \min_{x \in \chi^i_{c_{k'}}} |y_{k,s} - x\lambda_s|^2 \ge \sum_{k', d_{free}} \min_{x \in \chi^i_{\hat{c}_{k'}}} |y_{k,s} - x\lambda_s|^2\right) \tag{7}$$

where $\sum_{k', d_{free}}$ denotes that the summation is taken with index k' over d_{free} different values of k'.

Note that for binary codes and for the d_{free} points at hand, $\hat{c}_{k'} = \bar{c}_{k'}$. For the d_{free} bits let's denote

$$\begin{aligned} \tilde{x}_{k,s} &= \arg\min_{x \in \chi^i_{c_{k'}}} |y_{k,s} - x\lambda_s|^2 \\ \hat{x}_{k,s} &= \arg\min_{x \in \chi^i_{\bar{c}_{k'}}} |y_{k,s} - x\lambda_s|^2. \end{aligned} \tag{8}$$

It is easy to see that $\tilde{x}_{k,s} \ne \hat{x}_{k,s}$ since $\tilde{x}_{k,s} \in \chi^i_{c_{k'}}$ and $\hat{x}_{k,s} \in \chi^i_{\bar{c}_{k'}}$ where $\chi^i_{c_{k'}}$ and $\chi^i_{\bar{c}_{k'}}$ are complementary sets of constellation points within the signal constellation set χ. Also, $|y_{k,s} - x_{k,s}\lambda_s|^2 \ge |y_{k,s} - \tilde{x}_{k,s}\lambda_s|^2$ and $x_{k,s} \in \chi^i_{c_{k'}}$.

For convolutional codes, due to their trellis structure, d_{free} distinct bits between any two codewords occur in consecutive trellis branches. Let's denote d such that d_{free} distinct bits occur within d consecutive bits. The bit interleaver can be designed such that d consecutive bits are mapped onto distinct symbols (interleaver design criterion 1). This guarantees that there exist d_{free} distinct pairs of $(\tilde{x}_{k,s}, \hat{x}_{k,s})$, and d_{free} distinct pairs of $(x_{k,s}, \hat{x}_{k,s})$. The PEP can be rewritten as

$$P(\underline{c} \to \hat{\underline{c}}|H) = P\left(\sum_{k, d_{free}} |y_{k,s} - \tilde{x}_{k,s}\lambda_s|^2 - |y_{k,s} - \hat{x}_{k,s}\lambda_s|^2 \ge 0\right) \tag{9}$$

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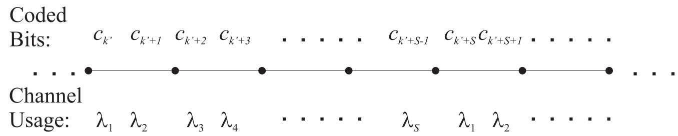

Fig. 1. Interleaver Design. Consecutive coded bits are transmitted over different subchannels as shown on a trellis path.

$$\leq P\left(\beta \geq \sum_{k,d_{free}} |\left(x_{k,s}-\hat{x}_{k,s}\right)\lambda_s|^2\right)$$
$$\leq Q\left(\sqrt{\frac{d_{min}^2\sum_{s=1}^{S}\alpha_s|\lambda_s|^2}{2N_0}}\right) \quad (10)$$

where

$\beta = \sum_{k,d_{free}} \beta_{k,s}$,

$\beta_{k,s} = (\hat{x}_{k,s}-x_{k,s})^*\lambda_s^* n_{k,s} + (\hat{x}_{k,s}-x_{k,s})\lambda_s n_{k,s}^*$,

α_s : denotes how many times the s^{th} subchannel is used

$\sum_{s=1}^{S}\alpha_s = d_{free}$.

For given H, $\beta_{k,s}$s are independent zero-mean Gaussian random variables with variance $2N_0|(\hat{x}_{k,s}-x_{k,s})\lambda_s|^2$. Consequently, β is a Gaussian random variable with zero mean and variance $2N_0\sum_{k,d_{free}}|(\hat{x}_{k,s}-x_{k,s})\lambda_s|^2$.

If the interleaver is designed such that the consecutive coded bits of length equal to the interleaver depth are transmitted on the same subchannel, then the performance is dominated by the worst singular value. In other words, the error event on the trellis occurs on the consecutive branches spanned by the worst subchannel, and $\alpha_S = d_{free}$. This results in a diversity order of $(N-S+1)(M-S+1)$ as in uncoded multiple beamforming as will be explained more detail below. However, the interleaver can be designed such that the consecutive coded bits are transmitted on different subchannels as shown in Figure 1 (interleaver design criterion 2). Criterion 2 guarantees that $\alpha_s \geq 1$, for $s = 1, 2, \ldots, S$.

Using an upper bound for the Q function $Q(x) \leq (1/2)e^{-x^2/2}$, PEP can be upper bounded as

$$P(\underline{c}\rightarrow\hat{\underline{c}}) = E\left[P(\underline{c}\rightarrow\hat{\underline{c}}|H)\right]$$
$$\leq E\left[\frac{1}{2}\exp\left(-\frac{d_{min}^2\sum_{s=1}^{S}\alpha_s|\lambda_s|^2}{4N_0}\right)\right]. \quad (11)$$

Let's denote $\alpha_{min} = \min\{\alpha_s : s = 1, 2, \ldots, S\}$. Then,

$$\frac{\sum_{s=1}^{S}\alpha_s|\lambda_s|^2}{S} \geq \frac{\alpha_{min}\sum_{s=1}^{S}|\lambda_s|^2}{S} \geq \frac{\alpha_{min}\sum_{s=1}^{N}|\lambda_s|^2}{N}. \quad (12)$$

Note that,

$$\Theta \triangleq \sum_{s=1}^{N}|\lambda_s|^2 = \|H\|_F^2 = \sum_{n,m}|h_{n,m}|^2 \quad (13)$$

is a chi-squared random variable with $2NM$ degrees of freedom (the elements of H, $h_{n,m}$, are complex Gaussian random variables). Using (11), (12), and (13) the PEP is upper bounded by

$$P(\underline{c}\rightarrow\hat{\underline{c}}) \leq E\left[\frac{1}{2}\exp\left(-\frac{d_{min}^2\alpha_{min}S}{4N_0N}\Theta\right)\right]. \quad (14)$$

The expectation in (14) is taken with respect to Θ with pdf $f_\Theta(\theta) = \theta^{(NM-1)}e^{-\theta/2}/2^{NM}(NM-1)!$ [13]. Consequently,

$$P(\underline{c}\rightarrow\hat{\underline{c}}) \leq \frac{1}{2^{NM+1}}\left(\frac{d_{min}^2\alpha_{min}S}{4N_0N}+\frac{1}{2}\right)^{-NM} \quad (15)$$
$$\approx \frac{1}{2^{NM+1}}\left(\frac{d_{min}^2\alpha_{min}S}{4N^2}SNR\right)^{-NM} \quad (16)$$

for high SNR. As can be seen from (16) the diversity order of BICMB at high SNR is NM. Consequently, BICMB achieves full diversity order independent of the number of spatial streams transmitted.

A very low complexity decoder for BICM can be implemented as in references [14], [15]. The same decoder can be used for BICMB as well: Instead of using the single-input single-output (SISO) channel value of BICM-OFDM for the decoder ([14] and [15]), one should use λ_s. Hence, BICMB provides a full spatial multiplexing, full diversity, and easy-to-decode system.

IV. Simulation Results

In the simulations below, the industry standard 64 states 1/2 rate (133,171) $d_{free} = 10$ convolutional code is used. Coded bits are separated into different streams of data and a random interleaver is used to interleave the bits in each substream. The interleaver guarantees the transmission of consecutive coded bits over consecutive subchannels. The coded bits are mapped onto symbols using 16 QAM with Gray labeling. Each packet has 1000 bytes of information bits, and the channel is changed independently from packet to packet.

Figure 2 illustrates the results for BICMB when 2 streams of data are transmitted at the same time. As can be seen from the figure, while transmitting at a spatial multiplexing of 2, BICMB achieves full diversity at high SNR for the 2×2, 3×3, and 4×4 cases.

Results of BICMB when spatial multiplexing is 3 for the 3×3, and 4×4 cases are given in Figure 3. A comparison of the 2×2, 3×3, and 4×4 cases, with full spatial multiplexing in each case, is given in Figure 4. Even though the 4×4 system transmits twice the data rate of 2x2 system, the performance of 4×4 system is significantly better than the the 2×2 system. This is due to the fact that the 4×4 system achieves a diversity order of 16 where the 2×2 system has a diversity order of 4. Consequently, BICMB provides both advantages of MIMO systems: It provides full diversity and full spatial multiplexing.

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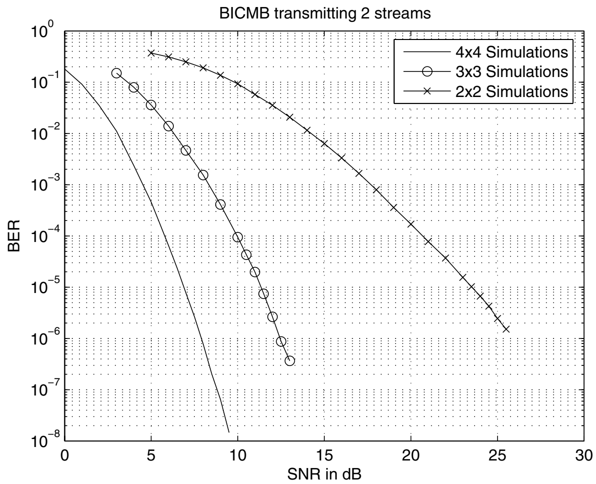

Fig. 2. BICMB transmitting 2 streams with the 2×2, 3×3, and 4×4 cases.

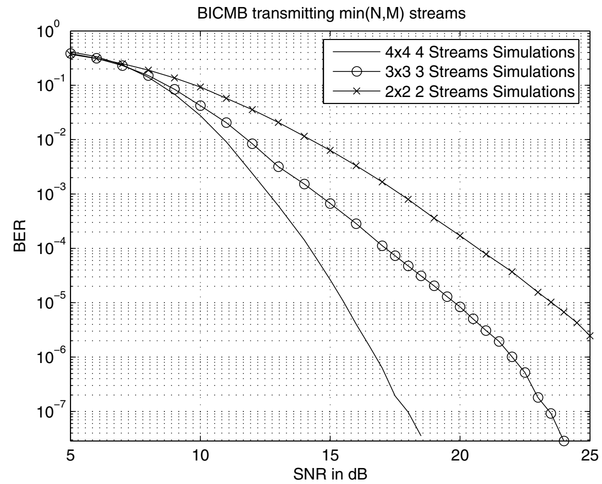

Fig. 4. BICMB transmitting $\min(N, M)$ streams with the 2×2, 3×3, and 4×4 cases.

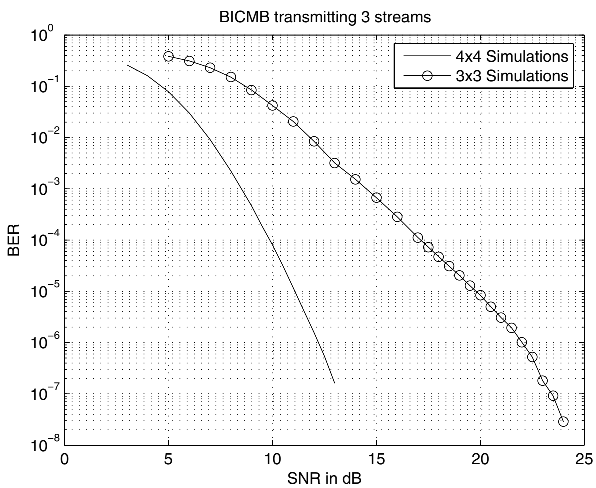

Fig. 3. BICMB transmitting 3 streams with the 3×3, and 4×4 cases

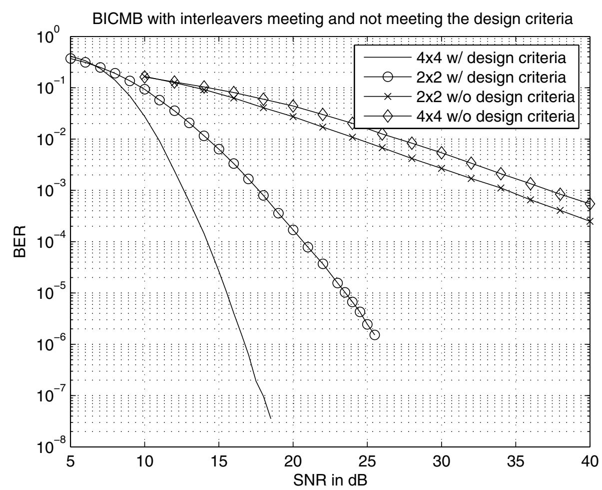

Fig. 5. BICMB transmitting $\min(N, M)$ streams with the 2×2, and 4×4 cases using an interleaver meeting and not meeting the design criteria.

Figure 5 illustrates the importance of the interleaver design. We simulated a random interleaver such that consecutive coded bits are transmitted over the same subchannel. In other words, on a trellis path, consecutive bits of length $1/S$th of the coded packet size are transmitted over the same subchannel. Consequently, an error on the trellis occurs over the paths that are spanned by the worst channel and the diversity order of coded multiple beamforming approaches to that of uncoded multiple beamforming.

Figures 6 and 7 show the simulation results of BICMB when compared to maximum likelihood decoder (MLD), minimum mean squared error receiver (MMSE), and zero forcing receiver (ZF) for spatial multiplexing of 2 and spatial multiplexing of 4 case, respectively. While MLD achieves a high diversity order with substantial complexity, ZF achieves a diversity order of $M - N + 1$ [16], [17]. MLD is known as the optimal receiver for a spatial multiplexing system. Using BICM at the transmitter with an interleaver spreading the consecutive bits over the transmit antennas and deploying MLD at the receiver end can be considered as the Vertical Encoding (VE) in [16]. Such a system is capable of providing a high diversity order. However, its substantially high complexity makes it almost impossible to implement. Therefore, suboptimal (therefore poorer performance) but easy-to-implement receivers are designed such as MMSE, ZF, successive cancellation (SUC) and ordered SUC [16]. As illustrated for the 2x2 case, at 10^{-5} bit error rate (BER), BICMB outperforms MLD by 4.5dB, while the performance gain compared to MMSE and ZF is more than 25dB. It is possible that the base station (or the access point) has more antennas than the receiver. BICMB with 4 transmit and 2 receive antennas with spatial multiplexing of 2 outperforms MLD by 15.5dB. When spatial multiplexing is 4, BICMB outperforms MMSE and ZF by more than 30dB at 10^{-5} BER. In order to achieve these

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performance gains, BICMB requires perfect CSI both at the transmitter and receiver. In some scenarios, e.g. fast fading environments, it may be difficult to provide the transmitter side with perfect CSI.

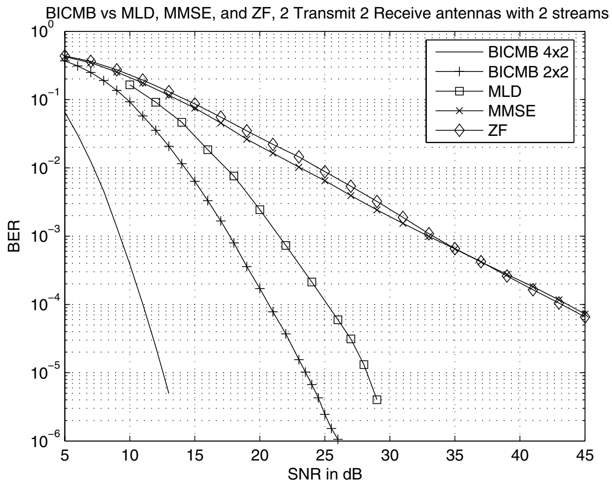

Fig. 6. BICMB vs MLD, ZF and MMSE for the 2×2 case.

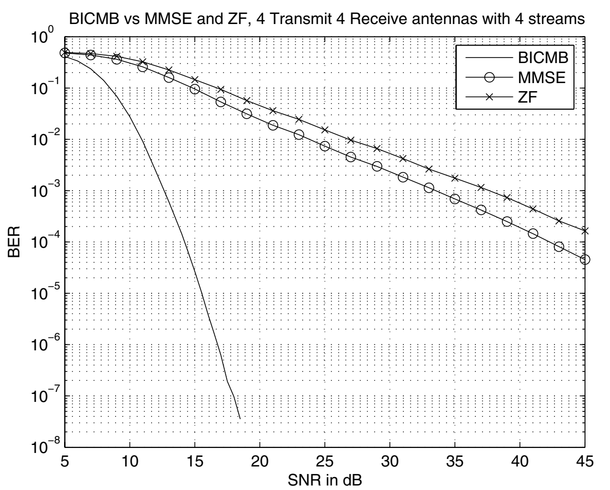

Fig. 7. BICMB vs ZF and MMSE for the 4×4 case.

We also simulated BICMB using the interleaver given in [18]. The interleaver in [18] satisfies the design criteria described in Section II. Hence, the performance of the interleaver in [18] is the same as the interleaver we used in our simulations.

V. CONCLUSION

In this paper we introduced bit interleaved coded multiple beamforming (BICMB). BICMB utilizes channel state information at the transmitter and the receiver. By doing so, BICMB achieves full spatial multiplexing of $\min(N, M)$, while maintaining full spatial diversity of NM for a $N \times M$ system. We presented interleaver design guidelines to guarantee full diversity at full spatial multiplexing.

Simulation results also showed that, exploiting the perfect channel knowledge, BICMB outperforms the optimal high complexity MLD receiver by more than 4dB, and outperforms easy-to-implement MMSE and ZF receivers by more than 25dB.

We described a low complexity implementation to decode BICMB systems. Overall, BICMB provides an easy-to-implement, high throughput (full spatial multiplexing) system, which maintains full spatial diversity and provides substantial performance gain.

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