

Low-Complexity Maximum Likelihood Detection of Orthogonal Space-Time Block Codes

Luay Azzam and Ender Ayanoglu

Center for Pervasive Communications and Computing
Department of Electrical Engineering and Computer Science
The Henry Samueli School of Engineering
University of California, Irvine
email: lazzam@uci.edu, ayanoglu@uci.edu

Abstract—In this paper, we propose a low complexity Maximum Likelihood (ML) decoding algorithm for orthogonal space-time block codes (OSTBCs) based on the real-valued lattice representation and QR decomposition. We show that for a system with rate $r = K/T$, where K is the number of transmitted symbols per T time slots; the proposed algorithm decomposes the original complex-valued system into a parallel system represented by $2K$ real-valued components, thus allowing for a simple and independent detection of the real and imaginary parts of each complex transmitted symbol. We further show that for a system employing the well-known Alamouti OSTBC and 16-QAM modulation scheme, the new approach achieves $> 82\%$ reduction in the overall complexity compared to conventional ML detection. Moreover, we show that for square L -QAM constellations, the proposed algorithm reduces the decoding computational complexity from $\mathcal{O}(L)$ for conventional ML to $\mathcal{O}(\sqrt{L})$ without sacrificing the performance.

I. INTRODUCTION

Space-time block codes (STBCs) from orthogonal designs (OSTBCs) are attractive since they achieve the maximum diversity, the maximum coding gain, and the highest throughput [1]. These codes are used in multiple-input multiple-output (MIMO) systems to introduce high capacity gains [2]. Their design allows for simple Maximum Likelihood (ML) decoding. The decoding complexity is very critical for practical employment of MIMO systems. Therefore, in OSTBCs proposed by Alamouti [3] and Tarokh et al. [4], each transmitted symbol is decoded separately, resulting in linear decoding complexity. Alamouti presented a simple transmit diversity technique for wireless communications with two transmit antennas, whereas, Tarokh et al. applied the theory of orthogonal designs to create space-time block codes for more than two transmit antennas [4].

For N transmit antennas, a complex orthogonal space-time block code is described by a $T \times N$ transmission matrix $\mathcal{G}_N$, where each entry in $\mathcal{G}_N$ is a linear combination of the K variables $s_1, s_2, \dots, s_K$ and their conjugates [5]. $\mathcal{G}_N$ can send K symbols from a signal constellation in a block of T channel uses. Since T time slots are used to transmit K symbols, the rate of $\mathcal{G}_N$ is defined as $r = K/T$ [6].

In this paper, we focus on the decoding complexity of OSTBCs. We introduce a new decoding algorithm for square QAM constellations based on QR decomposition of the real-valued

lattice representation and show that conventional simple ML detection can be further simplified. In other words, we show that the optimal ML performance for OSTBCs is obtained with a substantial reduction in the decoding complexity. We also compare our decoding complexity with that of conventional ML decoding.

The remainder of this paper is organized as follows: In Section II, we specify the system model and define the problem. In Section III, we introduce the new decoding algorithm and give few illustrative examples. A complexity discussion is provided in Section IV. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL AND PROBLEM DEFINITION

Consider a MIMO system with N transmit and M receive antennas, and an interval of T symbols during which the channel is constant. The received signal is given by

$$Y = \mathcal{G}_N H + V \quad (1)$$

where $Y = [y_t^j]_{T \times M}$ is the received signal matrix of size $T \times M$ and whose entry y_t^j is the signal received at antenna j at time t , $t = 1, 2, \dots, T$, $j = 1, 2, \dots, M$; $V = [v_t^j]_{T \times M}$ is the noise matrix; and $\mathcal{G}_N = [g_t^i]_{T \times N}$ is the transmitted signal matrix whose entry g_t^i is the signal transmitted at antenna i at time t , $t = 1, 2, \dots, T$, $i = 1, 2, \dots, N$. The matrix $H = [h_{i,j}]_{N \times M}$ is the channel coefficient matrix of size $N \times M$ whose entry $h_{i,j}$ is the channel coefficient from transmit antenna i to receive antenna j . The entries of the matrices H and V are independent, zero-mean, and circularly symmetric complex Gaussian random variables of unit variance.

Assuming that the channel H is known at the receiver, the ML estimate is obtained at the decoder by performing $\min_{\mathcal{G}_N} \|Y - \mathcal{G}_N H\|_F^2$, where $\|\cdot\|_F$ is the Frobenius norm. OSTBCs have a very simple and decoupled ML decoding algorithm. The squared norm $\|Y - \mathcal{G}_N H\|_F^2$ can be decoupled into K parts, where each part decodes one transmitted symbol independently [5]. We illustrate this by an example. Consider the OSTBC proposed by Alamouti [3] for $N = 2$ and defined as

$$\mathcal{G}_2 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}. \quad (2)$$

The receiver decodes s_1 and s_2 by decomposing the measure $\|Y - \mathcal{G}_N H\|_F^2$ into two parts, and minimizes each separately over all possible values of s_1 and s_2 that belong to the constellation used. Let the square L-QAM alphabet be given as Ω^2 , where $\Omega = \{-\sqrt{L}+1, -\sqrt{L}+3, \dots, \sqrt{L}-1\}$. Then, ML is equivalent to [4]

$$\hat{s}_1 = \arg \min_{s_1 \in \Omega^2} \left| \left[\sum_{j=1}^M \left(y_1^j h_{1,j}^* + (y_2^j)^* h_{2,j} \right) \right] - s_1 \right|^2 + \left(-1 + \sum_{j=1}^M \sum_{i=1}^2 |h_{i,j}|^2 \right) |s_1|^2$$

and,

$$\hat{s}_2 = \arg \min_{s_2 \in \Omega^2} \left| \left[\sum_{j=1}^M \left(y_1^j h_{2,j}^* - (y_2^j)^* h_{1,j} \right) \right] - s_2 \right|^2 + \left(-1 + \sum_{j=1}^M \sum_{i=1}^2 |h_{i,j}|^2 \right) |s_2|^2.$$

Obviously, ML detection in [5] is simple since it decodes each transmitted symbol independently. A number of ML decoders for $N > 2$ were derived in [5]. In a similar way, it was shown that the decoder decomposes the ML measure into K parts where each is minimized over all constellation points to decode one symbol separately. As a result, the complexity of ML decoding in [5] is $\mathcal{O}(L)$ which is linear with the constellation size L . Thus, the decoding algorithm can be implemented using only linear processing at the receiver. We will show in this paper that the complexity can still be reduced substantially. The algorithm proposed in this paper reduces the decoding complexity from $\mathcal{O}(L)$ to $\mathcal{O}(\sqrt{L})$ with a substantial reduction in the number of arithmetic operations required.

III. PROPOSED ALGORITHM

We start by rewriting (1) in matrix form

$$\begin{bmatrix} y_1^1 & \cdots & y_1^M \\ y_2^1 & \cdots & y_2^M \\ \vdots & \ddots & \vdots \\ y_T^1 & \cdots & y_T^M \end{bmatrix} = \mathcal{G}_N H + \begin{bmatrix} v_1^1 & \cdots & v_1^M \\ v_2^1 & \cdots & v_2^M \\ \vdots & \ddots & \vdots \\ v_T^1 & \cdots & v_T^M \end{bmatrix} \quad (3)$$

where

$$H = \begin{bmatrix} h_{1,1} & \cdots & h_{1,M} \\ h_{2,1} & \cdots & h_{2,M} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,M} \end{bmatrix}.$$

Then, we arrange the matrices Y , H , and V , each in one column vector by stacking their columns one after the other [7] to obtain

$$\begin{bmatrix} y_1^1 \\ \vdots \\ y_T^M \end{bmatrix} = \check{\mathcal{G}}_N \begin{bmatrix} h_{1,1} \\ \vdots \\ h_{N,M} \end{bmatrix} + \begin{bmatrix} v_1^1 \\ \vdots \\ v_T^M \end{bmatrix}$$

where $\check{\mathcal{G}}_N \triangleq I_M \otimes \mathcal{G}_N$, with I_M is the identity matrix of size M and $\otimes$ denoting the Kronecker matrix multiplication [7]. In the sequel, we assume that $M = 1$ for simplicity (we drop the superscript for $M = 1$ and consider $M \neq 1$ in illustrative examples subsequently). Then (3) can be rewritten as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} = \mathcal{G}_N \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ \vdots \\ h_{N,1} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_T \end{bmatrix}. \quad (4)$$

We specify the complex transmitted symbols $s_1, s_2, \dots, s_K$ of $\mathcal{G}_N$ by their real and imaginary parts as $s_i = x_{2i-1} + jx_{2i}$ for $i = 1, 2, \dots, K$. Then, we decompose the T -dimensional complex problem defined by (4) to a $2T$ -dimensional real-valued problem by applying the real-valued lattice representation defined in [8] to obtain

$$\check{y} = \check{H}x + \check{v}$$

or equivalently

$$\begin{bmatrix} \Re(y_1) \\ \Im(y_1) \\ \vdots \\ \Re(y_T) \\ \Im(y_T) \end{bmatrix} = \check{H} \begin{bmatrix} \Re(s_1) \\ \Im(s_1) \\ \vdots \\ \Re(s_K) \\ \Im(s_K) \end{bmatrix} + \begin{bmatrix} \Re(v_1) \\ \Im(v_1) \\ \vdots \\ \Re(v_T) \\ \Im(v_T) \end{bmatrix}. \quad (5)$$

The real-valued fading coefficients of $\check{H}$ are defined using the complex fading coefficients $h_{i,j}$ from transmit antenna i to receive antenna j as $h_{2l-1} = \Re(h_{l,1})$, and $h_{2l} = \Im(h_{l,1})$ for $l = 1, 2, \dots, N$ (recall that we restricted ourselves to $M = 1$, and therefore we only consider $j = 1$). Now, since $\mathcal{G}_N$ is an orthogonal matrix and due to the real-valued representation of the system using (5), we observe that

- All columns of $\check{H} = [\check{h}_1 \check{h}_2 \dots \check{h}_{2K}]$ where $\check{h}_i$ is the i th column of $\check{H}$, are orthogonal to each others, or equivalently

$$\langle \check{h}_i, \check{h}_j \rangle = 0, i \neq j \quad (6)$$

- The norm of every column in $\check{H}$ is equal to the norm of any other column in $\check{H}$, i.e.,

$$\|\check{h}_i\| = \|\check{h}_j\|, \quad i, j = 1, 2, \dots, 2K. \quad (7)$$

These two properties have a major impact on the complexity reduction of our proposed decoding algorithm (this will become clearer throughout the examples given below).

Applying QR decomposition to (5), we have

$$\begin{aligned} \check{y} &= QRx + \check{v} \\ Q^H \check{y} &= Rx + Q^H \check{v} \\ \bar{y} &= Rx + \bar{v} \end{aligned} \quad (8)$$

where $\bar{v}$ and $\check{v}$ have the same statistical properties since Q is unitary and so is Q^H . Recall that $\check{H}$ is a $2T \times 2K$ matrix. Then Q^H is a $2K \times 2T$ matrix and $\bar{y}$ is a vector of size $2K$. Since $\check{H}$ is an orthogonal matrix, QR decomposition produces

a $2K \times 2K$ diagonal R matrix (see [9] for proof), a property which substantially reduces the decoding complexity.

Using (8), the ML problem is now simpler and rather than minimizing $\|Y - \mathcal{G}_N H\|_F^2$, the solution is obtained by minimizing the metric $\|\bar{y} - Rx\|_2^2$ over all different combinations of the vector x . In other words, the ML solution is found by minimizing

$$\left\| \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_{2K} \end{bmatrix} - \begin{bmatrix} r_{1,1} & 0 & \cdots & 0 \\ 0 & r_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{2K,2K} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2K} \end{bmatrix} \right\|_2^2 \quad (9)$$

over all combinations of $x \in \Omega^{2K}$. This can be further simplified as

$$\hat{x}_i = \arg \min_{x_i \in \Omega} |\bar{y}_i - r_{i,i} x_i|^2 \quad (10)$$

for $i = 1, 2, \dots, 2K$. Then, the decoded message is

$$\hat{x} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{2K})^T.$$

This means that the ML solution is obtained by decoding the real and imaginary parts of each complex transmitted symbol independently through a simple 1×1 real-valued channel matrix. Note that this simplification is obtained through the observation of the orthogonality properties of $\check{H}$, and the QR decomposition in (8), resulting in (9) and (10). Obviously, this approach makes finding the ML solution simpler, and requires a smaller number of computational operations compared to conventional ML. To further clarify our proposed decoding algorithm, we now provide some representative examples.

Example 1: We consider the Alamouti OSTBC defined by (2) with $N = K = T = 2$ and $M = 1$. The received signal is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \quad (11)$$

Representing (11) in the real domain, we have

$$\begin{bmatrix} \Re(y_1) \\ \Im(y_1) \\ \Re(y_2) \\ \Im(y_2) \end{bmatrix} = \check{H} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \Re(v_1) \\ \Im(v_1) \\ \Re(v_2) \\ \Im(v_2) \end{bmatrix} \quad (12)$$

where $x_1 = \Re(s_1)$, $x_2 = \Im(s_1)$, $x_3 = \Re(s_2)$, $x_4 = \Im(s_2)$ and

$$\check{H} = \begin{bmatrix} h_1 & -h_2 & h_3 & -h_4 \\ h_2 & h_1 & h_4 & h_3 \\ h_3 & h_4 & -h_1 & -h_2 \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}.$$

Note that the channel matrix $\check{H}$ is orthogonal. Thus, applying QR decomposition to (12) and pre-multiplying both sides with Q^H produce

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \\ \bar{y}_4 \end{bmatrix} = \begin{bmatrix} r_{1,1} & 0 & 0 & 0 \\ 0 & r_{2,2} & 0 & 0 \\ 0 & 0 & r_{3,3} & 0 \\ 0 & 0 & 0 & r_{4,4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \\ \bar{v}_4 \end{bmatrix}.$$

The ML decoder finds $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4$ by performing

$$\hat{x}_i = \arg \min_{x_i \in \Omega} |\bar{y}_i - r_{i,i} x_i|^2$$

for $i = 1, 2, 3, 4$, and the decoded message is $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T$.

Example 2: For $M = 2$, we consider the OSTBC appeared in [5] for $N = 3$, $T = 8$, and $K = 4$ given by

$$\mathcal{G}_3 = \begin{pmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \end{pmatrix}^T.$$

The received signal at antennas 1 and 2 can be written as

$$Y = \mathcal{G}_3 H + V$$

or in matrix form as

$$\begin{bmatrix} y_1^1 & y_1^2 \\ \vdots & \vdots \\ y_8^1 & y_8^2 \end{bmatrix} = \mathcal{G}_3 \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \\ h_{3,1} & h_{3,2} \end{bmatrix} + \begin{bmatrix} v_1^1 & v_1^2 \\ \vdots & \vdots \\ v_8^1 & v_8^2 \end{bmatrix}. \quad (13)$$

To obtain the real-valued representation of (13), we arrange the matrices Y , H and V , each in one column vector by stacking their columns one after the other [7] as

$$\begin{bmatrix} y_1^1 \\ \vdots \\ y_8^1 \\ y_1^2 \\ \vdots \\ y_8^2 \end{bmatrix} = \check{\mathcal{G}}_3 \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ h_{3,1} \\ h_{1,2} \\ h_{2,2} \\ h_{3,2} \end{bmatrix} + \begin{bmatrix} v_1^1 \\ \vdots \\ v_8^1 \\ v_1^2 \\ \vdots \\ v_8^2 \end{bmatrix}. \quad (14)$$

where $\check{\mathcal{G}}_3 \triangleq I_2 \otimes \mathcal{G}_3$, with I_2 is the identity matrix of size 2 ($M = 2$) and $\otimes$ denoting the Kronecker matrix multiplication.

Now, decomposing (14) into its real-valued representation, we have

$$\begin{bmatrix} \Re(y_1^1) \\ \Im(y_1^1) \\ \vdots \\ \Re(y_8^2) \\ \Im(y_8^2) \end{bmatrix} = \check{H} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_8 \end{bmatrix} + \begin{bmatrix} \Re(v_1^1) \\ \Im(v_1^1) \\ \vdots \\ \Re(v_8^2) \\ \Im(v_8^2) \end{bmatrix} \quad (15)$$

and the 32×8 real-valued channel matrix $\check{H}$ is given by

$$\begin{bmatrix} h_1 & -h_2 & h_3 & -h_4 & h_5 & -h_6 & 0 & 0 \\ h_2 & h_1 & h_4 & h_3 & h_6 & h_5 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_7 & -h_8 & h_9 & -h_{10} & h_{11} & -h_{12} & 0 & 0 \\ h_8 & h_7 & h_{10} & h_9 & h_{12} & h_{11} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & h_{11} & h_{12} & -h_9 & -h_{10} & -h_7 & -h_8 \\ 0 & 0 & h_{12} & -h_{11} & -h_{10} & h_9 & -h_8 & h_7 \end{bmatrix}$$

where h_i , $i = 1, 3, \dots, 11$ and h_j , $j = 2, 4, \dots, 12$ are the real and imaginary parts of $h_{1,1}, h_{2,1}, h_{3,1}, h_{1,2}, h_{2,2}, h_{3,2}$.

Applying QR decomposition to (15) and pre-multiplying

both sides by Q^H , we obtain

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_8 \end{bmatrix} = \begin{bmatrix} r_{1,1} & 0 & \cdots & 0 \\ 0 & r_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{8,8} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_8 \end{bmatrix} + \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \vdots \\ \bar{v}_8 \end{bmatrix}$$

and the ML solution is obtained by carrying out

$$\hat{x}_i = \arg \min_{x_i \in \Omega} |\bar{y}_i - r_{i,i}x_i|^2$$

for $i = 1, 2, \dots, 8$. Then, the decoded message is $\hat{x} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_8)^T$.

Generally, we observe that the proposed algorithm produces $2K$ parallel 1×1 real-valued subsystems for any OSTBC, thus making the detection of the real and imaginary parts of each transmitted complex symbol be carried out independently. This results in a simplified ML problem that can be solved in a parallel fashion to obtain the optimal solution while substantially reducing the overall decoding complexity.

IV. COMPUTATIONAL COMPLEXITY

In this section, we compare the computational complexity of our proposed algorithm with that of conventional ML detection. The overall complexity is measured in terms of the number of operations required to decode the transmitted signals for each block period T . A complex multiplication is equivalent to 4 real multiplications R_M and 2 real additions R_A , while a complex addition is equivalent to 2 real additions. We split the complexity formula into two parts in order to represent R_M and R_A independently. We denote the complexity of our proposed algorithm by $\mathcal{C}_{PR}$, and show it as a two dimensional vector where the first dimension is the number of real multiplications and the second, the number of real additions, then

$$\mathcal{C}_{PR} = 2K\sqrt{L}(4R_M, 2R_A). \quad (16)$$

Note that performing QR decomposition requires additional number of computations. Due to the special structure of the channel matrix $\tilde{H}$, QR can be simplified into two simple steps. To illustrate this, let $\tilde{H} = [\tilde{h}_1 \ \tilde{h}_2 \ \dots \ \tilde{h}_{2K}]$ where $\tilde{h}_i$ is the i th column of $\tilde{H}$. Then, due to (6), R is diagonal. The definition of the diagonal elements in R in QR decomposition is $r_{i,i} = \|\tilde{h}_i\|$. Due to (7) the matrices Q and R are computed by

Step 1: Calculate the diagonal elements of the matrix R by finding $r_{1,1} = \text{norm}(\tilde{h}_1)$ and then set $r_{i,i} = r_{1,1}$ for $i = 2, \dots, 2K$. (Note that due to (7) all diagonal elements are equal).

Step 2: Calculate the unitary matrix $Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_{2k} \end{bmatrix}$ where $q_i = \tilde{h}_i / r_{i,i}$ for $i = 1, 2, \dots, 2K$.

Finding R requires $2MT + 12 R_M$ and $2MT - 1 R_A$, and computing Q requires $16MTK R_M$, assuming that a square root operation and a real division are equivalent to 12 and 4 real multiplications respectively [10].

Moreover, the computation of $\bar{y} = Q^H \tilde{y}$ requires $4MTK R_M$ and $4MTK - 2K R_A$. Therefore, (16) is rewritten taking into account the complexity of computing QR and $Q^H \tilde{y}$ as

$$\mathcal{C}_{PR} = (MT(20K + 2) + 12 + 8K\sqrt{L})R_M, \\ (MT(4K + 2) + 2K(2\sqrt{L} - 1) - 1)R_A. \quad (17)$$

Conventional ML detection [5], on the other hand, performs simple detection for each complex symbol independently. The complexity $\mathcal{C}_{ML}$ can be derived using the presentation in [5] (see Appendix of [5] for details) as

$$\mathcal{C}_{ML} = L((4MN(T + K) + 12K)R_M, \\ (4MN(T + K) + 6K)R_A). \quad (18)$$

We give a comparison between $\mathcal{C}_{PR}$ and $\mathcal{C}_{ML}$ in terms of the number of real multiplication and real additions considering $N = 2, 3, 4$ for different constellation sizes. In Table I, we show this comparison for $N = 2$ considering the Alamouti OSTBC defined in (2). In Table II, we show the same comparison for 4×1 and 3×2 systems using the OSTBCs $\mathcal{G}_4$ and $\mathcal{G}_3$ defined in [5].

Clearly, the complexity gain obtained by the proposed algorithm is substantial. Finally, it is important to emphasize the fact that the complexity reduction, as shown in all tables, becomes greater as L is larger.

TABLE I

OF REAL MULTIPLICATIONS AND REAL ADDITIONS VS L FOR 2×1 SYSTEM USING ALAMOUTI CODE

		L	4	16	64	256
R_M	ML		224	896	3584	14336
	PR		128	160	224	352
R_A	ML		176	704	2816	11264
	PR		31	47	79	143

TABLE II

OF REAL MULTIPLICATIONS AND REAL ADDITIONS VS L FOR 4×1 AND 3×2 SYSTEMS USING $\mathcal{G}_4$ AND $\mathcal{G}_3$

		L	4	16	64	256
4×1	R_M	ML	960	3840	15360	61440
		PR	732	796	924	1180
	R_A	ML	864	3456	13824	55296
		PR	167	199	263	391
3×2	R_M	ML	1606	6424	25696	102784
		PR	1388	1452	1580	1836
	R_A	ML	1248	4992	19968	79872
		PR	311	343	407	535

V. CONCLUSIONS

An efficient ML decoding algorithm based on QR decomposition of the channel matrix is proposed for orthogonal space-time block codes. The performance is shown to be

optimal while reducing the decoding complexity significantly compared to conventional ML. Furthermore, we show that the complexity of this algorithm is $\mathcal{O}(\sqrt{L})$ compared to $\mathcal{O}(L)$ for conventional ML, and consequently the complexity gain becomes grater as the constellation size is larger.

REFERENCES

- [1] H. Jafarkhani, "Space-Time Coding: Theory and Practice," 2005.
- [2] N. Al-Dhahir, C. Fragouli, A. Stamoulis, W. Younis, and A. Calderbank, "Space-Time Processing for Broadband Wireless Access," *IEEE Communications Magazine*, vol. 40, pp. 136–142, Sep. 2002.
- [3] S. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. on Selected Areas in Communications*, vol. 16, pp. 1451–1458, Oct. 1998.
- [4] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.
- [5] —, "Space-Time Block Coding for Wireless Communications: Performance Results," *IEEE J. on Selected Areas in Communications*, vol. 17, pp. 451–460, Mar. 1999.
- [6] X. Liang, "A High-Rate Orthogonal Space-Time Block Code," *IEEE Trans. Commun.*, vol. 7, pp. 222–223, May 2003.
- [7] H. Gamal and M. Damen, "Universal Space-Time Coding," *IEEE Trans. Inf. Theory*, vol. 49, pp. 1097–1119, Jun. 2003.
- [8] L. Azzam and E. Ayanoglu, "Reduced Complexity Sphere Decoding for Square QAM via a New Lattice Representation," *IEEE GLOBECOM*, pp. 4242–4246, Nov. 2007.
- [9] G. Golub and C. V. Loan, "Matrix Computations," 3rd Edition, *The John Hopkins University Press, Baltimore*, 1996.
- [10] S. Yatawatta, A. Petropulu, and C. Graff, "Energy-Efficient Channel Estimation in MIMO Systems," *EURASIP Journal on Wireless Communications and Networking*, pp. 1–11, Dec. 2005.